Maximizing Investment Income of SSP for Spectrum Trading in Cognitive Radio Networks

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Abstract-More and more researches have demonstrated the benefits of cognitive radio technology in improving flexibility and efficiency of spectrum utilization. In order to encourage primary users (PUs) to share their idle spectrum resources with secondary users (SUs), spectrum trading frameworks are developed. In this paper, the investment problem of spectrum service provider (SSP) is considered which obtains spectrum from PUs and provides service to multiple SUs. The SUs' actions are estimated according to statistical data. A estimation method for channels number is proposed basing on maximizing the SSP's investment income. A Markov chain model is used to analyze the SSP's state transition and calculate the SU's waiting time and queuing size by queuing theory. The optimal number of channels is deduced with marginal analysis theory. In either spectrum purchase or auction, the SSP could adjust its investment strategy timely and flexibly according to these parameters.

I. INTRODUCTION

Traditional fixed spectrum allocation and usage mode has blocked development of wireless communication technologies. Currently cognitive radio (CR) is regarded as a revolution in breaking the barrier [1]. More and more researches have demonstrated the benefits of CR technology in improving flexibility and efficiency of spectrum utilization. In a CR network, the secondary users (SUs) can opportunistically access the licensed spectrum of primary users (PUs). To encourage PUs share their idle spectrum with SUs, researchers propose some spectrum trading mechanisms, which transform spectrum from resources to goods [2][3][4]. The PUs sell the use rights of licensed spectrum for a period of time, meanwhile the SUs could achieve available spectrum at certain costs.

The large organizational SUs are relatively easy to get spectrum from PUs. They need more spectrum and are glad to pay higher costs for monopolizing the use rights. But for the small organizational or personal SUs, it is difficult to participate in the spectrum trading directly due to limitations of heterogeneity, complex trading rules, restricted spectrum requirements and unequal informations. These SUs are called as end-users. An effective solution is to develop some knowledgeable and professional spectrum service providers (SSPs) to trade with PUs and achieve vacant spectrum. Spectrum with large bandwidth is divided into different channels. When the end-users expect to work using spectrum, they apply for channels from a SSP. The SSPs provide an easy, efficient and equitable platform in spectrum tradings between PUs and endusers. The operation mode among PUs, a SSP and end-users is similar to a cellular network in some ways, and the role of SSP approximates to a base station.

Obviously the SSP faces a significant dilemma: If it obtains less spectrum, the available channels it could provide to endusers are less. The deferred spectrum service perhaps causes loss of end-users who turn to other SSPs in their communication regions. On the other hand, if the SSP purchases too many spectrum goods but the channel selling is disappointing, the investment of SSP will fail. And worse still, the spectrum will be wasted once again.

Some studies use game theory to model the interactions among PUs, SSPs and end-users [5][6][7]. In these papers, the end-users are regarded as a whole game participator to influence the SSP. However, the end-user's spectrum requests come gradually in stead of together in fact. Every request influences the system state as an individual behaviour. This paper use a Markov chain model to analyze the SSP's state transformation with the end-users coming gradually. The waiting time and queuing size are calculated with queuing theory. The optimal number of channels for SSP to earn the most investment income is deduced with marginal analysis.

Our contributions are as follows:

• A Markov chain model and queuing theory are introduced into the analysis of SSP's states. The optimal number of channels is calculated according to marginal analysis. The results of optimal channel number and cost-performance ratio data have guiding significance for the SSP to participate in either spectrum direct purchases or auctions.

• The attention on spectrum utilization lasts after resource reallocation, instead of ending formerly. And the user experience is concerned. If the utilization ratio is often below 80%, the spectrum is not taken full advantage of. And if the customer churn is often above 10%, the user experience is terrible. The optimal number of channels proposed in this paper achieves desired balance among utility, spectrum utilization and end-user experience.

The rest of paper is organized as follows: We introduce our system model in section II. Section III does some theoretical analysis to the model where we give the method to calculate its parameters. In section IV, we evaluate our system with



Fig. 1. Spectrum from PUs to SSP



Fig. 2. An Application Example of SSP and End-users

simulation experiments. Finally, Section V summarizes our conclusions.

II. SYSTEM MODEL

A. Spectrum Model and Spectrum Access Method

The SSP acquires the use rights of spectrum segments in a time span and divides them into channels as shown in Fig. 1. Fig. 2 illustrates the end-users request spectrum service from a SSP.

Likened to cellular networks, the communication range of a SSP is not as large as its interference range, so its channels must be different from other adjacent ones. The SSP allocates different end-users different channels.

The division of channels could be with single or multiple bandwidth types. Single bandwidth division is easy to calculate and operate. But the SSP can't provide various spectrum services on diverse bandwidth requests, and gets the most efficient spectrum utilization hardly. The SSP with multiple bandwidth channels provides more flexible services to endusers. On the other hand, a lot of statistics and analysis are necessary before division. This paper focuses on the optimal number of channels to maximize SSP's investment income, so the single bandwidth division is adopted to simplify the complexity caused by multiple bandwidth.

B. System Assumption

In our assumptions, spectrum demands of different endusers are independent from each other. The SSP divides spectrum into n channels, and every channel has the same bandwidth to each other. The channels allocated to different end-users are different.

It is assumed that the SSP gains the income w from an end-user if it provides an available channel to him in a unit of time, meanwhile pays h to PU for getting the spectrum use right. There are two things to note: Firstly, it is necessary for a SSP to operate normally that w > h. Secondarily, the values of w and h are generally not constant due to spectrum market fluctuations.

When the process of end-users acquiring channels from a SSP is modeled, three factors should be taken into count: arrival of spectrum requests, service time of each request, and queueing rules.

Denote the end-users arriving in interval [0,t] as X(t), so the arrival of spectrum requests is modeled by a poisson process:

$$P\{X(t_0+t) - X(t_0) = k\} = e^{-\lambda t} \frac{(\lambda t)^k}{k!}$$
(1)

where λ denotes the average arrival rate. The mathematical expectation of arriving end-users in interval t is $\lambda \times t$.

Then the service time is considered. If the average departure rate is μ , an exponential distribution is used to describe its density:

$$f(t') = \mu e^{-\mu t'} \qquad (t' > 0). \tag{2}$$

In CR network studies, such models are employed widely. For instance, spectrum sharing about MAC protocols [8][9], MAC-layer sensing schemes [10][11], and adaptive spectrum sensing framework [12] all have mentioned.

Finally the queuing rule is assumed as First Come First Served (FCFS). When there are no channels available to allocate, the SSP will delay service responses to the coming end-users. Consequently, the end-users queue to wait the previous ones to release channels. But perhaps the queuing end-users will terminate their requests and turn to other SSPs in the communication scope for spectrum service. Wherefore the current SSP will lose some potential incomes.

Obviously, the possibility of that an end-user ends service request has a connection with his waiting time. For simplicity, the intensity of end-user departure α_k is assumed in proportion to k, when n channels of the SSP are all busy and there are k end-users queuing. Totally, there are n + k end-users in the system. It can be credited as:

$$\alpha_k = k\delta \qquad (0 < \delta < 1),\tag{3}$$

where δ is the departure factor.

At the same time the channel use is assumed to be independent from each other.

III. PROPOSED ALGORITHM

A. Estimating Arrival Rate

Just like described in previous sections, the difference between the SSP network structure and a cellular or WiFi network is mainly in the spectrum access mode, and the traffic patterns of end-users are similar. So the estimation of poisson



Fig. 3. Autocorrelation of Arrival Rate in a real WiFi Network

distribution parameter is illustrated with real WiFi network data from CRAWDAD [13].

The statistics of end-user arrivals are regarded as time series. The autocorrelation coefficients of the series are calculated with:

$$\rho(t, t+k) = \frac{E[X(t) - EX(t)][X(t+k) - EX(t+k)]}{\sqrt{DX(t)DX(t+k)}}$$
(4)

which are shown in Fig. 3.

It is observed that the autocorrelation coefficient reaches the peak at every 24 hours. It can be associated with natural law of human life immediately. Therefore the cycle of end-user arrivals can be determined as 24 hours. During the estimation of end-user actions, the parameter λ_i is considered unaltered in one hour, and another new parameter λ_{i+1} is used in next hour. The parameters in a day is denoted as series $\{\lambda_1, \lambda_2, \ldots, \lambda_{24}\}$.

 λ_i is estimated with the sample average of end-user arrivals:

$$\hat{\lambda_i} = \frac{1}{m} (x_i + x_{24+i} + \dots + x_{(m-1)*24+i}) \quad (m > 1) \quad (5)$$

where m is the number of sampling periods. $\hat{\lambda}_i$ is the uniformly unbiased estimate of λ_i .

B. Estimating Service Time

Unlike the arrival rate, the end-user's service time is not periodic. So the average service time is estimated with another method, namely the maximum likelihood method:

$$\hat{\mu} = \frac{N_{est}}{\sum\limits_{i=-N_{est}}^{-1} t'_i}.$$
(6)

In (6), N_{est} is the number of end-users whose service time is used to estimate μ , and t'_i is the service time of *i*th end-user who finishes communication before the estimation.

C. Calculating Optimal Channels Number with Markov Chain

When the number of end-users exceed the available channels, the queuing end-users will depart with probability α_k . It is obvious that the queue size should be limited for practical reasons, such as system capacity and impatient end-users. The queuing end-user departure rate $\alpha_k = k\delta$ denotes that the queuing size k is not more than $\frac{1}{\delta}$.

If *n* denotes the channels amount of SSP, let *u*, where $u = 1, 2, \dots, n, \dots$, be the number of coming end-users, including being allocated channels, queuing for allocation and leaving impatiently. A random variable S_v is defined to stand for the state that there are *v* end-users in the system with $v = 0, 1, \dots, n, \dots, n + \frac{1}{\delta}$. Then a Markov chain model is introduced to calculate $Pr\{S_v = v\}$ which denotes the probability mass function of S_v . Thus, the end-users coming process is modeled by a Markov chain, $\{S_v\}$, having finite $n + \frac{1}{\delta}$ states theoretically. The number of end-users in the system is represented by the variable in circle, as shown in Fig. 4.

The transition probability of Markov chain is denoted by $q_{i,j}$ which can be written as

$$q_{i,j} \stackrel{\triangle}{=} Pr\{S_{v+1} = j | S_v = i\} \\ = \begin{cases} \lambda, & j = i+1, \\ i\mu, & j = i-1, i \le n, \\ n\mu + \alpha_k, & j = i-1, n < i \le n + \frac{1}{\delta}, \\ 0, & others. \end{cases}$$
(7)

where $i, j = 0, 1, \dots, n, n+1, \dots, n+\frac{1}{\delta}$. The probability transition matrix for Markov chain S_v is denoted by Q which is derived as follows:

$Q \stackrel{\scriptscriptstyle riangle}{=} \{$	$[q_{i,j}]$	}								
	0	λ	0	0	• • •	• • •	0	• • •		0]
	μ	0	λ	0	•••	•••	0	• • •		0
	0	2μ	0	λ	•••	•••	0	• • •	•••	0
	:	÷	÷	÷	·	÷		÷	÷	:
=	0	0	• • •	$n\mu$	0	λ	• • •	• • •	•••	0 .
	0	0	• • •	0 n	$\mu + \alpha_1$	0	λ	• • •	•••	0
	0	0	•••	•••	$0 \ n\mu$	$\iota + \alpha_2$	0	•••	•••	0
	:	÷	÷	÷	÷	÷	÷	÷	÷	:
	0	0			•••	•••		n	$\mu \mu + 1$	0
	-									(8)

The probability $Pr\{S_v = v\}$ is equivalent to the *v*-steps transition probability from the state of 0 to *v*, which can be expressed as

$$Pr\{S_v = v\} = Q^v|_{(0,v)}.$$
(9)

For simplicity, p_v is used to denote $Pr\{S_v = v\}$. In balance state, the Kolmogorov-Chapman equation is:

$$\begin{cases} \lambda p_{0} = \mu p_{1}, \\ \lambda p_{1} = 2\mu p_{2}, \\ \dots \\ \lambda p_{n-1} = n\mu p_{n}, \\ \lambda p_{n} = (n\mu + \alpha_{1})p_{n+1}, \\ \lambda p_{n+1} = (n\mu + \alpha_{2})p_{n+2}, \\ \dots \\ \lambda p_{n+\frac{1}{\delta}-1} = (n\mu + 1)p_{n+\frac{1}{\delta}}. \end{cases}$$
(10)



Fig. 4. Markov Chain for State Changes of SSP

It can be gotten that:

$$p_{v} = \begin{cases} \frac{(n\rho)^{v}}{v!} p_{0}, & 0 \le v \le n, \\ \frac{(n\rho)^{n}}{n!(1+b)(1+2b)\cdots[1+(v-n)b]} p_{0}, n < v \le n + \frac{1}{\delta}. \end{cases}$$
(11)

Among above expression, $\rho = \frac{\lambda}{n\mu}$, $b = \frac{\delta}{n\mu}$. In balance state, the regularity condition is established:

$$\sum_{v=0}^{n+\frac{1}{\delta}} p_v = 1.$$
 (12)

So the solution about p_0 is:

$$p_{0} = \{\sum_{v=0}^{n} \frac{(n\rho)^{v}}{v!} + \sum_{v=n+1}^{n+\frac{1}{\delta}} \frac{(n\rho)^{n}}{n!(1+b)(1+2b)\cdots[1+(v-n)b]}\}$$
(13)

The average queuing size is:

$$L_w = \sum_{k=0}^{n+\frac{1}{\delta}} k p_{n+k}.$$
 (14)

The mean value of working channels equals to the mean number of end-users who have been allocated channel, which is:

$$L_s = \overline{v} = \sum_{v=0}^{n-1} v p_v + \sum_{v=n}^{n+\frac{1}{\delta}} n p_v.$$

$$(15)$$

In a unit of time, the SSP gains income w from an end-user for providing an available channel to him, meanwhile pays hto PU. So the investment income is:

$$U(n) = wL_s - hn. (16)$$

Based on marginal analysis, the number of channels which brings about the most utility for the SSP, namely n^* , is:

$$\begin{cases} U(n^*) > U(n^* - 1), \\ U(n^*) > U(n^* + 1). \end{cases}$$
(17)

By combing Eqs. (16) and (17), it is obtained that:

$$\begin{cases} wL_s(n^*) - hn^* > wL_s(n^* - 1) - h(n^* - 1), \\ wL_s(n^*) - hn^* > wL_s(n^* + 1) - h(n^* + 1). \end{cases}$$
(18)

Solving Eq. (18) for $\frac{h}{w}$ yields:

$$L_s(n^*+1) - L_s(n^*) < \frac{h}{w} < L_s(n^*) - L_s(n^*-1).$$
 (19)

If the SSP purchases spectrum from some certain PUs and the prices are known in advance, only n^* is unknown in Eq. (19). According to Eq. (15), the values of $L_s(n^*)$ with different n^* are calculated to make up the different intervals $(L_s(n^*+1) - L_s(n^*), L_s(n^*) - L_s(n^*-1))$. The proper n^* contributes the interval within which $\frac{h}{w}$ lies. Such a procedure is demonstrated in Section IV. In the spectrum market the SSP seeks spectrum with $w_0 \times n^*$ bandwidth from one or more PUs at the cost of $h \times n^*$.

Another possibility is that the SSP attends spectrum auctions in which the prices are floating. Both n^* and h are indeterminate, so it's a linear indeterminate equation with two changeable unknowns. On the contrary, the sold price w should be fixed relatively to form a stable spectrum platform. The SSP has to choose proper number of channels n^* and price hfor equality. Although the lower prices sustain more channels, QoS has also to be considered.

It is worth noting that the actual bandwidth of spectrum acquired by the SSP is uncertain probably because of resource inequality. However the optimal number of channels provides a practice direction to guide the spectrum investment.

IV. EXPERIMENT AND EVALUATION

In this section the estimation of end-user arrival is validated according to experimental data from a real WiFi network [13] firstly.

In Fig. 5, the arrival of beginning two years are sample data to estimate the future arrival, and the estimation result is compared with latter real data. It is derived from the comparison that the estimation reflects the periodic trend of end-user arrivals.

Since effective and actual data are lacking, the simulation experiment is adopted to verify the service time estimation. In Fig. 6, when the simulated service time average changes, the estimated curve fluctuates with it. It indicates that the proposed method reflects service time changes properly, although there is a certain delay. It also shows that the estimation is more accurate on condition that service time is smooth relatively.

Subsequently, the investment income maximization approach is validated with simulation experiments. The buying and selling prices are known, and the changeable factor is channel number. The adopted simulation parameters are shown in Tab.I.

Firstly we calculate $L_s(n^* + 1) - L_s(n^*) \sim L_s(n^*) - L_s(n^* - 1)$ and get the values as shown in Tab. II. Then $\frac{h}{w}$ is calculated to be 0.67. Compared with Tab. II, the result lies in the range (0.5492, 0.8845) and corresponds to the third line. So the optimal number of channels is identified as 28.

In order to check the optimal approach, 18, 23, 33 and 35 channels are selected as the conditions of simulation comparison experiments. With different channels, the income



Fig. 5. Evaluation of Arrival Rate Estimation with Real Data



Fig. 6. Evaluation of Service Time Estimation with Simulation

TABLE I PARAMETERS OF SIMULATION

Average arrival rate	$\lambda = 20$
Average service time	$\mu = 0.67 hour$
Investment per channel	h = 2yuans
Income per channel	w = 3yuans
Invested channels	18,23, 28,33, 35

TABLE II VALUES OF n^* and $L_s(n^*)$

n^*	$L(n^*)$	$L(n^*+1) - L(n^*) \sim L(n^*) - L(n^*-1)$
26	26.9995	$0.9890 \sim 0.9995$
27	27.9884	$0.8845 \sim 0.9890$
28	28.8729	$0.5492 \sim 0.8845$
29	29.4221	$0.2297 \sim 0.5492$
30	29.6518	$0.0941 \sim 0.2297$

and utilization of SSP are studied. The income means the return on investment, and the utilization could test the effective spectrum usage in reallocation.

As shown in Fig. 7, with the end-users coming, the income of 28 channels increases more obviously than other scenes. It is the double of 18 channels, 50% more than 23 channels, and 120% more than 33 channels. The income of 38 channels is the least (approximately zero), although the channels number



Fig. 7. Utility of SSP with Diverse Channels



Fig. 8. Spectrum Utilization Ratio of SSP with Diverse Channels



Fig. 9. Waiting Time of End-users with Diverse Channels

is 10 more (about 35.7%) than the optimal one. Such a result indicates that the number of channels is not the more the better. For the SSP, too many channels mean vacant and consumptive, and it brings pecuniary loss.

Comparing spectrum utilization with different channels in Fig. 8, it is found that at the beginning of 18 or 23 channels experiments the spectrum utilization reaches 100% quickly and has kept all along. It is because the channel supply is not adequate to spectrum demands, so there are always end-



Fig. 10. Customer Churn with Diverse Channels

users waiting in the system and almost all channels have been employed. The poignant contrast is that in the system with 33 or 38 channels, the utilization has never reached 100%. Actually it floats around 75% in 33 channels system and 65% in 38 channels system. It means the channels supply exceeding demands in these systems and there are channels always vacant. In the optimal 28 channels system the spectrum utilization achieves 100% in more than three quarters time and the lowest is about 70% in a certain time.

The spectrum utilization experiment results show that too little channels make the SSP lacking spectrum resource and many end-users leave because of waiting too long. Oppositely too many channels exceeding actual requests waste the spectrum resource obviously and such a situation deviates from the original intention of cognitive radio technology.

Fig. 9 and Fig. 10 reflect the end-user experience with different numbers of channels. The customer churn is the ratio of the leaving customers without any services to the total customers. When the channels are 33 or 38, the corresponding average waiting time and customer churn are both zero. The channel number reduces to 28, the average waiting time fluctuates from 0 to 20 hours and customer churn is about 4%. Considerated with the total working time and arrival rate, such a situation is acceptable. When the channels are 23, the waiting time increases to 35 hours around and customer churn is 30%, where the operating state of SSP is serious. With the channel number reducing to 18, the waiting time is about 50 hours and customer churn is 50% or so, which means the half of coming end-users leave without service. The SSP provides bad spectrum service experience to end-users with less number of channels, although it reduces spectrum waste.

According to all the results above, the small channel amount brings higher utilization rate but worse end-user experience, and the big one is just opposite. The optimal channel number proposed in this paper achieves desired balance among income, spectrum utilization and end-user experience.

V. CONCLUSION

In this paper we pay attention to the investment income of SSP and spectrum usage in resource reallocation. A way to estimate end-user actions according to statistical data and a method to calculate the optimal parameters of system are introduced. At last the empirical data of a real WiFi network and simulation experiments are used to evaluate the approaches. The results prove that in a cognitive network our system perform evenly in utility, utilization and end-user experience.

Our future work is to analyse and value the proposed system in the whole spectrum trade from the PUs to the SSP, then to the end users. Both the cooperative and the noncooperative game may exist in the trade process. The establishment of global equilibrium will bring more conditions to the SSP for spectrum reallocation probably.

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REFERENCES

- Ian F. Akyildiz, Won-Yeol Lee, Mehmet C. Vuran, Shantidev Mohanty, NeXt generation/dynamic spectrum access/cognitive radio wireless networks: a survey, Computer Networks, Volume 50, Issue 13, pp. 2127-2159, 15 September 2006.
- [2] Zhou X., and Zheng H., TRUST: A general framework for truthful double spectrum auctions, in the 28th Conference on Computer Communications (INFOCOM 2009), pp. 999-1007, April 2009.
- [3] Lin Chen, S. Iellamo, M. Coupechoux, P. Goodlewski, An Auction Framework for Spectrum Allocation with Interference Constraint in Cognitive Radio Networks, in the 29th Conference on Computer Communications (INFOCOM 2010), pp. 1-9, March 2010.
- [4] G. S. Kasbekar, S. Sarkar Spectrum auction framework for access allocation in cognitive radio networks, IEEE/ACM Transactions on Networking (TON), Volume 18 Issue 6, pp. 1841-1854, December 2010.
- [5] D. Niyato, E. Hossain, and Z. Han, Dynamics of multiple-seller and multiple-buyer spectrum trading in cognitive radio networks: a gametheoretic modeling approach, IEEE Trans. Mobile Comput., vol. 8, no. 8, pp. 1009-1022, 2009.
- [6] L. Duan, J. Huang, and B. Shou, Investment and pricing with spectrum uncertainty: a cognitive operators perspective, IEEE Trans. Mobile Comput., vol. 10, no. 11, pp. 1590-1604, 2011.
- [7] L. Yang, H. Kim, J. Zhang, M. Chiang, and C. W. Tan, *Pricing-based decentralized spectrum access control in cognitive radio networks*, IEEE/ACM Transactions on Networking, vol. 21, no. 2, pp. 522-535, 2013.
- [8] Haythem A. Bany Salameh, Marwan Krunz, Ossama Younis, *Cooperative adaptive spectrum sharing in cognitive radio networks*, in IEEE/ACM Transactions on Networking, Vol. 18, Issue 4, August 2010.
- [9] Sanqing Hu, Yu-Dong Yao, Zhuo Yang, Cognitive medium access control protocols for secondary users sharing a common channel with time division multiple access primary users, Wireless Communications and Mobile Computing, 2012.
- [10] Sanqing Hu, Yu-Dong Yao, Zhuo Yang, MAC protocol identification approach for implement smart cognitive radio", 2012 IEEE International Conference on Communications Workshops, pp. 5608 - 5612, 2012.
- [11] Shoukang Zheng, Ying-Chang Liang, Pooi Yuen Kam, Anh Tuan Hoang, Cross-Layered Design of Spectrum Sensing and MAC for Opportunistic Spectrum Access, in Wireless Communications and Networking Conference (WCNC), pp. 1-6, April 2009.
- [12] D. Datla, R. Rajbanshi, A. M. Wyglinski, and G. J. Minden, parametricadaptive spectrum sensing framework for dynamic spectrum access networks, in Proceedings of the 2nd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN 2007), pp. 482-485, 2007.
- [13] Michael Lenczner, Benoit Gregoire, and F. Proulx, CRAWDAD trace set ilesansfil/wifidog/session (v. 2007-08-27), downloaded from http://crawdad.cs.dartmouth.edu/ilesansfil/wifidog/session.