Resource Allocation with Multiple Channel Width in Cognitive Cellular Networks

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***Abstract*—In this paper we introduce a novel evaluation criterion for spectrum allocation in cognitive cellular networks that is the utility of base-station. Our intention is to allocate the channels with different bandwidth to the end-users according to diverse Quality-of-Service (QoS), which not only increases the system's profits but also improves the actual spectrum utilization. We estimate the end-user's behaviors and model the spectrum demands basing on the statistical data in a past period. The multidimensional bounded knapsack problem is introduced to divide channels, of which the proposed balance between value density and request probability strategy gets the approximate solution. The simulation experiment results show its good performance both in the utility and the spectrum utilization of base-stations, especially when the resources are deficient.**

I. Introduction

Cellular networks have succeeded in the last twenty years, but they have to be put to the test in increasing rapidly data traffic. The popularity of smart-phones which is not so much the phones as the hand-held computers, integrates several multimedia information services. In an open market report of Ericsson, it is described that the data traffic of cellular networks has doubled from the third quarter 2011 to the the third quarter 2012, and mobile data traffic driven mainly by video is expected to grow with a CAGR (Compound Average Growth Rate) of around 50 percent in the time frame 2012-2018, which entails growth of around 12 times by the end of 2018 [1].

Since the objective of cognitive radio (CR) technology is to reallocate spectrum resources reasonably and efficiently, the combination of these two technologies brings the promising prospect [2]. The spectrum sharing in cognitive cellular (CGC) networks is a two-phase procedure. In the first phase, the CGC network gets spectrum resources from the PUs through sensing [3] and borrowing [4], or auctions [5][6][7].

In the second phase, the base-stations (BSs) of CGC network cells allocate the spectrum to the end-users. At present there is few studies on spectrum allocation in the CGC networks particularly, but some studies about the CR networks are developing. A distributed resource allocation method is proposed in [8], whose purpose is to balance the queue length and promote the fairness. But as we know, the architecture of CGC network is centralized, and the spectrum allocation from a BS to end-users is planned by the BS rather than the end-users voluntarily. The resource allocation standard in [9] is to maximize the coverage of CR network through power control method. Since in a CGC network the cell's range is certain, expanding the coverage doesn't absorb much attention.

Moreover, the above achievements are all concentrating on the technical property of CGC network. Since the economical benefits encourage the CGC network to value the end-user's requirements and devote itself to improve the QoS, it's the same important as the technical aspect. The better user experience is always one of the most significant goals of cellular networks. Such an absence is a shortage of the study on spectrum allocation in CGC networks.

The resources allocation in a CR network is summarized to a multidimensional knapsack problem and a greedy algorithm is introduced [10]. Such a process is instructive for us to consider the spectrum allocation in CGC networks. However, the allocation in CGC network is not only a Bounded Knapsack Problem but also a Multiple Knapsack Problem, and the mathematical model should be defined again firstly. Secondly, in order to adopt the mathematical expression above, the end-user actions should be modeled according to the empirical data analysis. Thirdly the density of value greedy algorithm only concerns the reward and cost that ignores the volatility of end-user demands, so we make an improvement on it.

Our contributions are as follows:

- The BS's utility in the second phase of two-phase spectrum sharing in a CGC network is discussed first, which has been ignored in previous studies. The appropriate economic stimulation will boost the CGC network to serve the end-users more attentively. The relevance between profit and spectrum also promotes the spectrum saving in resources reallocation.
- The balance between value density and request probability (BVDRP) algorithm is proposed. The existing heuristic algorithms can not completely apply to the multidimensional bounded knapsack problem. In BVDRP the pursuit for benefit, the cost of system and the irregular of end-user requests are all concerned.

The rest of paper is organized as follows: the system model is introduced in section II. Section III does some theoretical analysis to the model where we give the method to calculate the parameters of it. In section IV, the system is evaluated with simulation experiments. Finally, Section V summarizes our conclusions.

II. SYSTEM MODEL

A cellular network distributes over land areas called cells, each served by at least one fixed-location transceiver, known as a BS. A BS is the center of his local communication cell.

For clear and simple description, in this paper a BS's spectrum programme is discussed in his local cell, so relevant spectrum usage mode is simple. In a CGC network cell if a user wants to communicate with another, he has to get an available channel from the BS. The BS allocates different users different channels. A complete graph K_n can be used to describe the conflict of a BS's users. The communication range of a BS is not as large as the interference, so the spectrum channels of a BS must be different from other adjacent ones. In this paper we discuss the spectrum allocation of the single BS.

III. PROPOSED ALGORITHM

We denote the spectrum resources that the BS acquires from PUs by B. The system auctions or rents them from the PUs or in the secondary markets, so they are not continuous but a series of spectrum blocks B_i, which can be written as

$$B = \sum_{i=1}^{m} B_i \tag{1}$$

where $i = 1, 2, \cdots, m$.

The n distinct kinds of channel width the system provides to the end-users is denoted by w_j, where $j = 1, 2, \cdots, n$. The every actual scale of w_j could be decided by advance surveys or experimental data collections. In the paper [11], the authors describe the statistical results about the videos of YouTube whose bit rates are nearly the same in the website. Such a research proves that it is possible to fix on the width of similar applications preliminarily. So a suitable series of channel width $\{w_j\}$ could be confirmed according to several kinds of major applications.

Suppose the amount of demands for channel width w_j is s_j, for which the income is v_j in a unit of time. The pricing strategy is a market behavior and not formed by the technical decision.

The technical problems faced to in this economical procedure are following two: how to specify the demand quantity s_j for channel width w_j, and how to divide the spectrum blocks to available channels.

A. To specify demanded quantities

According to the CGC network's characteristics, the system only occupies the resources for a specific time, after which they have to be returned to PUs. At that time the system will rent some new blocks of spectrum. Such a process means that the spectrum resources are updated regularly, in which the period is called a spectrum period. The key to divide spectrum blocks suitably is to predict end-user actions and calculate s_j accurately in the following spectrum period.

Denote the spectrum period as Δt, the number of channel requests S is

$$S = \int_{t_0}^{t_0+\Delta t} f(t)\, dt \tag{2}$$

where $f(t)$ is the probability density function of arrival rate of end-user channel requests.

In CR network studies, the arrival of spectrum requests is described with a Poisson process usually [12][13], which can be defined as

$$P\{X(t'_0 + t') - X(t'_0) = k\} = e^{-\lambda t'} \frac{(\lambda t')^k}{k!} \tag{3}$$

where λ denotes the average arrival rate.

During the estimation of end-user actions, the parameter λ is considered unaltered in a time period $\Delta t'$, and another new parameter λ is used in the next period. Such a period is a natural period which is different from the spectrum period. The parameters of λ are denoted as series $\{\lambda_{t'}\}$. Considering the spectrum trading process, we regard that Δt is not less than $\Delta t'$ and expressed with

$$\Delta t = \Delta t_0 + \Delta t'_1 + \Delta t'_2 + \cdots \Delta t'_u + \Delta t_{u+1} \tag{4}$$

where Δt_0 is the time earlier than the natural period $\Delta t'_1$ and Δt_{u+1} is the time laster than $\Delta t'_u$.

Since $\{\lambda_{t'}\}$ are the average arrival series, we get

$$S \approx \frac{\Delta t_0}{\Delta t'_0}\lambda_0 + \lambda_1 + \cdots + \lambda_u + \frac{\Delta t_{u+1}}{\Delta t'_{u+1}}\lambda_{u+1}. \tag{5}$$

Hence the estimation of S is translated to estimate the average arrival $\{\lambda_{t'}\}$ which is considered as a discrete time series. For such a nonstationary time series including seasonal fluctuations and trend potentially, a $ARIMA(p, d, q)(P, D, Q)^s$ model is adopted. The expressional forms are

$$Y_t = (1 - B)^d (1 - B^s)^D, \tag{6}$$

$$\phi(B)\Phi(B^s)(1 - B)^d(1 - B^s)^D Y_t = \theta(B)\Theta(B^s)e_t, \tag{7}$$

where s is the seasonal periodicity. Meanwhile p is the autoregressive order, q is the moving average order, and d is the sum order. P, Q and D are corresponding seasonal parameters. B is the backward operation. In following section, an application example is given to illustrate the process.

After estimating the end-user requests S, the request ratio p_j for bandwidth w_j in all spectrum requests has to been considered. Just like $\{\lambda_{t'}\}$, the bandwidth ratio series $\{p_{jt'}\}$ in natural periods is modeled with $ARIMA(p', d', q')(P', D', Q')^s$. Then

$$p_j \approx \frac{\Delta t'_0 p_{j0} + \Delta t'_1 p_{j1} + \cdots + \Delta t'_u p_{ju}}{\Delta t}. \tag{8}$$

The expected demand for the channel with bandwidth w_j is

$$s_j = S * p_j, \qquad 1 \le j \le n. \tag{9}$$

However, S is the number that the system expects to provide. Usually it could not be realized because of the limitations of spectrum resource, the uncertainties of market behavior, as well as the irregularities of spectrum blocks. So the system has to try its best to obtain as many resources as it can. In such a case, it have to been considerd how to make full use of vested spectrum and maximize the system's utility. This involves the next question: how to divide the channels.

B. To divide channels

To divide m discontinuous spectrum blocks into n different channels, it's not only a Bounded Knapsack Problem (i.e., BKP) but also a Multiple Knapsack Problem (MKP). So we call it a Multiple Bounded Knapsack Problem (MBKP).

The decision problem form of the knapsack problem is NP-complete [14], thus it is expected that no algorithm can be both correct and fast (polynomial-time) on all cases. Our interest is rapid-speed and low-complexity solutions, so the greedy methods are concerned which devote to get the best choice basing on the current situation in spite of the overall conditions. These courses could get the satisfactory solution instead of the optimal one at the expense of a lot of time and space. Such heuristic strategies couldn't get the optimal solution necessarily, but achieve the desired objective quickly [10]. It is suitable for our system where the channel division is updated termly and the convenient acquisition is more important.

The key of greedy algorithm is to appoint the greedy strategy. The density of value greedy strategy sorts the items in decreasing order of value per unit of weight, $\frac{v_i}{w_i}$. In our scenario, the value stands for the system's profits from the end-users while the weight represents the provided bandwidth. So the density of value means the ratio of the item's reward to its cost. It is just the policy adopted in the paper [10]. Since the amount of spectrum resources the system owned is limited and the BS works in a commercial mode, the system wants to gain more input with less output. So the density of value greedy strategy is more suitable for channel division in the CGC networks than other developed ones.

However we have to consider the probabilities the end-user requests lie in the different spectrum periods. Though some channels' value densities are higher, they appear much less in the requests. Such idle channels not only bring few incomes for the system, but also waste the spectrum resources seriously.

On account of such considerations, an improved channel division strategy is adopted that is a balance between value density and request probability (BVDRP), whose space complexity is $O(n)$ and time complexity is $O(n \lg n)$.

Firstly we process the value density data, which perhaps are many times more than the probabilities. After being processed they could be in the same interval comparison. Denote the polished data by h_j which can be written as

$$h_j = \lg \frac{v_j}{w_j} \tag{10}$$

where $j = 1, 2, \cdots, n$.

Then we introduce the weighting factors, α and β, into the improved greedy algorithm, which are defined as

$$\alpha + \beta = 1, \qquad \alpha \geq 0, \beta \geq 0. \tag{11}$$

The determination coefficient of channel division decides the allocation order. The channel with greater coefficient will be allocated earlier, which can be expressed as

$$d_j = \alpha * h_j + \beta * p_j \tag{12}$$

where $j = 1, 2, \cdots, n$.

The mathematical model is to select the proper channel width from $\{w_j\}$, with which the system divides the m spectrum blocks to maximize the system's utility. If $c_{i,j}$ denotes the amount of channels which are divided from the spectrum block B_i with bandwidth w_j, such a problem can be described as

$$max \sum_{i=1}^{m} \sum_{j=1}^{n} d_j c_{i,j} \tag{13}$$

subject to

$$\sum_{j=1}^{n} w_j c_{i,j} \leq S_i, \qquad i = 1, 2, \cdots, m, \tag{14}$$

and

$$\sum_{i=1}^{m} c_{i,j} \leq t_j, \qquad j = 1, 2, \cdots, n. \tag{15}$$

The values α and β show the preference the system has between the value density and bandwidth request probability. Most often, they are values in the interval $(0, 1)$. Two extreme situations are as follows: When $\alpha = 1$ and $\beta = 0$, the system divides the channels following the density of value greedy strategy completely in spite of the request probability. It means the pursuit of maximum benefits and the assumption of vacant channels risk. So it is a radical channel division strategy. On the other hand when $\alpha = 0$ and $\beta = 1$, the system cares about the full use of resources and tries to maximize the channel request probabilities, where the anticipation for incomes is weak. Such a strategy is conservative.

In practical applications, the values of α and β are selected according to the system load, the balance between income and resource utilization, and other demands.

The algorithm to describe the BVDRP strategy, which takes into account both the value density and request probability, is shown as in Algorithm 1.

IV. EXPERIMENT AND EVALUATION

In view of end-user arrival rate, a cell of CGC network is similar to a WiFi network. Then we use real WiFi network data from CRAWDAD [15] to verify our prediction.

Firstly the autocorrelations of end-user arrival rate in previous 8000 hours in 2006 are calculated in which the 24th item is the most. It can be associated with the natural law of human life immediately. A seasonal $ARIMA(2, 0, 4)(1, 1, 1)^{24}$ model is chosen to predict the arrival rate of subsequent time. The estimation results is shown in Fig. 1, which reflects the real end-user arrival well.

Since the WiFi records don't include bandwidth, the lack of real data results in bandwidth request prediction absent. The more real-network data collection will be our future work.

Algorithm 1 Algorithm of BVDRP for channel division

Require:

The weighting factors α and β;
The bandwidth of every spectrum block S_i, $i = 1, 2, \cdots, m$;
The amount of every kind of provided channel t_k, $k = 1, 2, \cdots, n$;
The bandwidth of every kind of provided channel w_k, $k = 1, 2, \cdots, n$;

Ensure:

1: Compute processed density of value $h_j = \lg \frac{v_j}{w_j}$, $j = 1, 2, \cdots, n$;
2: Compute determination coefficients for provided channels $d_k = \alpha * h_k + \beta * p_k$, $k = 1, 2, \cdots, n$;
3: Initialize $i = 1$;
4: Assign $j = l$, when $d_l = \max\{d_k\}, k = 1, 2, \cdots, n$ and $t_k > 0$;
5: Divide a channel $c_{i,j}$ and $|c_{i,j}| = w_j$, when $t_j > 0$ and $|S_i| \geq w_j$;
6: Update $t_j \leftarrow t_j - 1$ and $S_i \leftarrow S_i - c_{i,j}$;
7: Update $d_j \leftarrow -\infty$, and $j \leftarrow l$ when $d_l = \max\{d_k\}, k = 1, 2, \cdots, n$, if $t_j = 0$;
8: Update $i \leftarrow i + 1$, if $|Si| < w_j$;
9: Goto step 5 if $i \leq m$;
10: **return** $c_{i,j}, i = 1, 2, \cdots$ and $j = 1, 2, \cdots$.

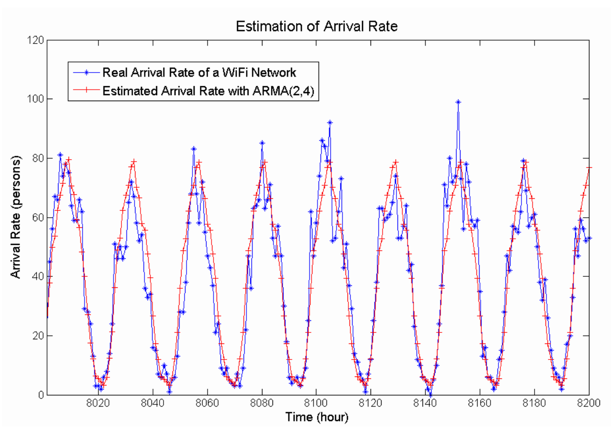

Fig. 1. Comparison of Arrival Rate Estimation with Real Data

Subsequently we validate the channel division approach this paper put forward with simulation experiments. The BVDRP is compared with other four strategies including the density of value greedy, the value greedy, the channel width from small to large, and the channel width from large to small. The reasons that they are selected are as follows:

• *Density of Value Greedy* : This strategy is introduced in the paper [10] as a desirable heuristic strategy. Compared with other previous ones, it's a practical method with regarding not only the reward but also the cost.

• *Value Greedy* : It highlights the purpose to seek the profits merely, which is the economical study on spectrum allocation in CGC networks superficially.

TABLE I
CHANNEL DIVISION SIMULATION PARAMETERS

Weighting factor	$\alpha = 0.5$, $\beta = 0.5$
Channel width specifications	$0.2MHz, 0.3MHz, 0.4MHz, 0.5MHz, 0.6MHz$
Price per channel	$2yuans, 2.9yuans, 3.7yuans, 4.4yuans, 5.0yuans$
Amount of channels expected to allocate	100
A single spectrum block maximum	$2MHz$
Total bandwidth of spectrum resources	$2 \sim 50MHz$
Poisson distribution parameters of request arrival	$\lambda = 100$
Exponential distribution parameters of service time	$\mu' = 1$
Gaussian distribution parameters of bandwidth request	$\mu = 350$, $\sigma^2 = 100$
Request sampling number	5000

• *Channel Width from Small to Large* : This strategy divides the channels as many as possible and accords with the objective of [3] to enlarge the networks capacity.

• *Channel Width from Large to Small* : It is the opposite to the third strategy above, so it is conducted for reference.

We show their comparisons in the channel allocation efficiencies, the utilities of system, and the spectrum utilizations.

The simulation experiment parameters are shown in Tab. 1. Suppose the end-user requests coming with a Poisson process whose parameter $\lambda = 100$, the service time following an Exponential distribution with $\mu' = 1$, and the bandwidth probability following a Gaussian distribution with $\mu = 350$ and $\sigma^2 = 100$. The weighting factors α is 0.5 and β is 0.5 too, which means the system values the value density and the request probability uniformly.

The spectrum blocks are generated randomly, the amount of which is from 2MHz to 50MHz increasingly. We divide the channels using five different allocation strategies and calculate the allocation efficiencies with allocated bandwidth divided by total bandwidth.

As shown in Fig. 2, the five division methods are no obvious difference in the allocation efficiencies. When the spectrum resources reach 40 MHz totally, the five curves begin to overlap. It illustrates that 40 MHz resources have satisfied the system's all demands, over which the allocation results are consistent in spite of division difference. The allocation curves decrease gradually over 40 MHz, because the superfluous resources are wasted after the satisfaction of spectrum demands.

According to above simulation results it is known that the different division strategies don't reflect differences if the spectrum resources acquired from primary users are sufficient.

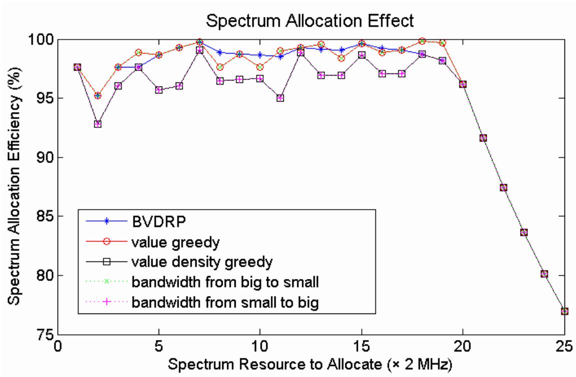

Fig. 2. Channel Division Effect

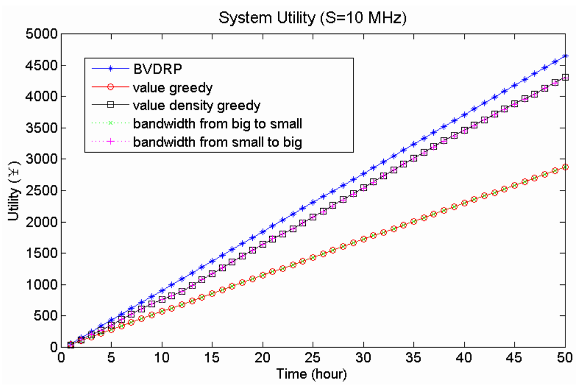

Fig. 3. System Utility when S=10

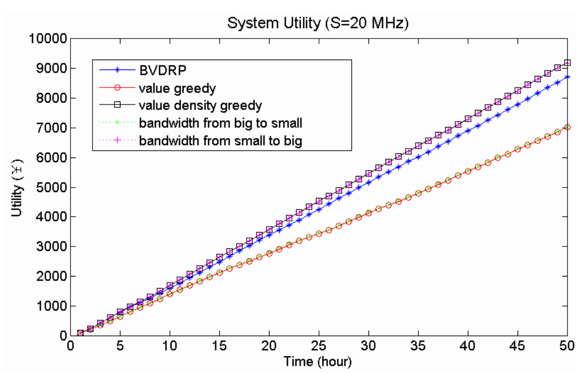

Fig. 4. System Utility when S=20

It is understood that the base-stations in CGC networks could not achieve a plenty of resources due to the unbalance. So we select the spectrum scenes of 10 MHz, 20 MHz and 30 MHz to compare the system utilities and the spectrum utilizations. Such scenes reflect the spectrum resources from exceedingly poor to relatively enough.

Fig. 3, 4 and 5 show that with different distributable resources the system utilities increase linearly following time growth. The incomes of the BVDRP, the density of value and the bandwidth from small to large strategies are approximate, which of the value greedy and the bandwidth from large to small are lower.

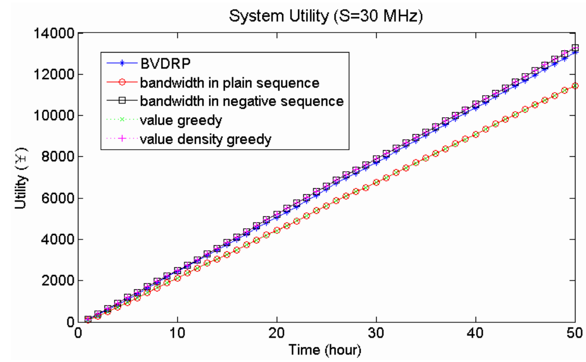

Fig. 5. System Utility when S=30

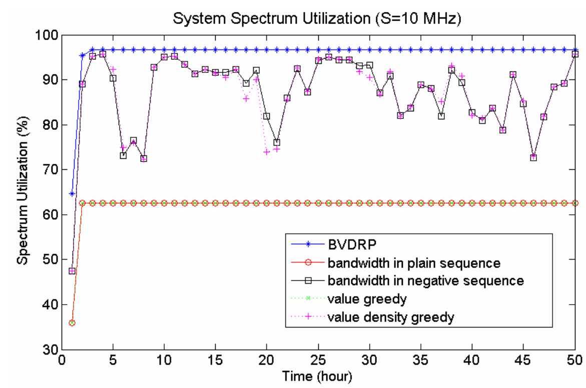

Fig. 6. Spectrum Utilization when S=10

The value greedy strategy pursues value maximization too much and ignores that more value means more bandwidth. On the face of seeking incomes, this division approach brings less utility to the system when the resources are limited. The bandwidth from large to small strategy wants to divide the channels in advance which need large bandwidth, until the allocation is difficult. Then the remainder spectrum is divided to smaller channels. Though the higher price is charged for the larger channel, the spectrum spent is more and the number of available channels is less. Finally the total utility of system is less.

Among the three channel division strategies producing more incomes, the density of value greedy wants to exchange less bandwidth for more profits. The bandwidth from small to large tries to divide more available channels when the spectrum resources are certain, and exactly in the experiments the smaller channels has the higher density of value. So they obtain more utilities for the system.

The BVDRP strategy this paper proposed takes into account both the value density and the request probability. Though its density of value is not the highest, for which the channel requests emerge more frequently. The divided channels are vacant rarely that brings higher total earnings. When the lack of spectrum resources is serious (S = 10 MHZ), the system obtains the most profits following this process.

Fig. 6, 7 and 8 show the spectrum utilizations using the five

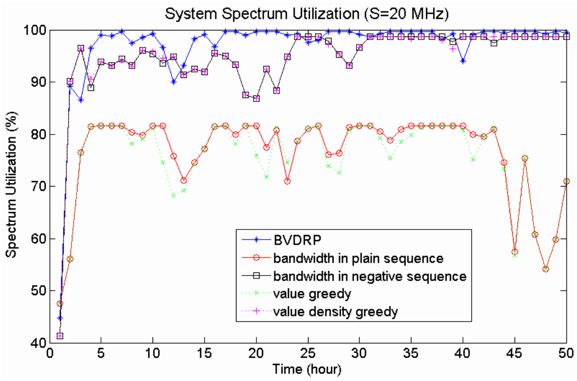

Fig. 7. Spectrum Utilization when S=20

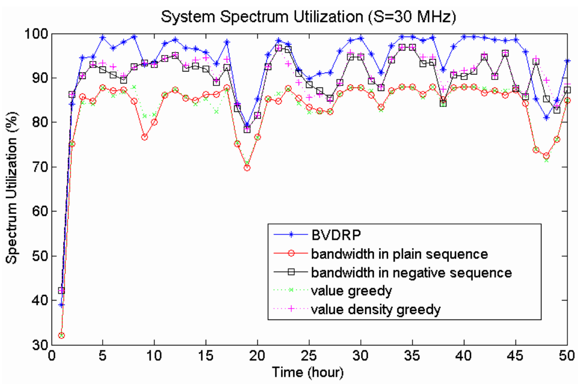

Fig. 8. Spectrum Utilization when S=30

channels division strategies.

When S = 10MHZ, the system have resources scarcely. The spectrum utilization rates are stable at around 63% with the value greedy and the bandwidth from large to small strategies. The usage of available channels divided by the density of value and the bandwidth from small to large strategies are affected according to the channel requests, the spectrum utilizations of which fluctuate strongly within the interval 70% to 95%. Our BVDRP strategy takes the full consideration of customer requests, so it takes better place in the spectrum utilization rate that is kept above 96% approximately.

When S=20MHz and S=30MHz, the available channels are more and more due to the spectrum resources are trending to abundance. So the customers queue is eased and there are vacant channels sometimes, which leads to the spectrum utilization rates reducing at some certain extents. They fluctuate from 68% to 88% with the value greedy and the bandwidth from large to small strategies, and from 76% to 97% with the density of value and the bandwidth from small to large ones. The BVDRP approach presents the highest utilization of five strategies, which is about from 76% to 99%.

V. Conclusion

In this paper we propose a channel division approach for the wireless service provider (the base-station) in CGC networks. The end-user arrival rate and bandwidth probability are estimated with ARIMA models. Then we adopt the balance between value density and request probability strategy (BVDRP) to solve the multiple bounded knapsack problem and divide the discontinuous spectrum blocks into channels. At last the method is evaluated with real data and simulation experiments. The results prove that in a CGC network our process will not only bring more profits to the service provider but also make better use of the spectrum resources. And the advantage is more obvious while the resource is less.

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