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Unified Particle-based Simulation of Deformable Solid-Fluid Interaction

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Abstract This paper presents a novel unified particle-based method to simulate deformable solid-fluid interaction based on SPH. In this method deformable solid-fluid interaction is treated as fluid-fluid interaction with large rest density ratio, thus the two-way interaction can be achieved directly by solving the governing equations of fluid. The density discontinuities at interfaces between deformable solid and fluid are handled. Based on a stable and accurate surface curvature calculation, a new approach to model surface tension is proposed so that the flow of fluid on solid and the formation of fluid droplets can be realistically simulated. The experimental results show that the model can simulate realistic deformable solid-fluid interaction.

Keywords deformable solid-fluid interaction · particle-based method · SPH

1 Introduction

Recently, realistic simulation of physical phenomena is attracting more and more attention. However, deformable solid-fluid interaction has not been well handled. Currently, coupled models[1, 2] are widely used in computer graphics, but the variety of the simulated materials and effects is often constrained by the interfaces between the models. It is highly desirable to have a uniform simulation model which can handle different types of materials and eliminate the need of defining an interface for coupling different models. Particle-based methods have become increasingly popular for the animation of elasto-plastic materials and fluids. Recently, in order to simplify interactions, the fully particle-based approaches[3, 4] have been put forward to simulate fluids, solids and phase transitions in a unified way.

This paper describes a novel unified particle-based method to simulate deformable solid-fluid interaction. The method treats deformable solid as a special fluid whose motion is constrained to elastic deformation, thus the mutual interaction can be achieved directly by solving the governing equations of fluid. We introduce a new density computation to handle interface discontinuities and consequently derive new formulations for pressure and viscous forces. To realistically simulate the flow of fluid on solid and the formation of fluid droplets, a new surface tension model which can obtain a stable and accurate scheme for surface curvature is provided.

2 Related Work

Deformable solid-fluid interaction is a typical issue in computer graphics. However, the solving approaches mainly focus on coupling models. Genevaux et al. [5] proposed a method to simulate the interaction between solids represented by mass-spring networks and an Eulerian fluid grid. But the nodes of a mass-spring network are not quite well suited for the application of interaction forces. Müller et al. [1] developed a method

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in which SPH fluid particles interact with Lagrangian meshes by adding boundary particles to the surface of the mesh. It does not guarantee the constant particle density near the wall boundary. To simulate the two-way interaction between the Eulerian fluid and Lagrangian elastic solid, Chentanez et al. [2] enforced coupling constraints by combining both the pressure projection and implicit integration steps into one set of simultaneous equations.

Recently, several particle-based systems which unify simulation algorithms for fluid, rigid solid and deformable solid have been proposed. Müller et al. [6] proposed a fully particle-based technique to model elastic, plastic and melting behavior of objects, where a Moving Least Squares (MLS) approach is used to calculate the elastic forces. Keiser et al. [3] merged the Navier Stokes equations with the equations for deformable solids to simulate solids, fluids and phase transitions in a uniform particle-based framework. Solenthaler et al. [4] use SPH to approximate the Jacobian of the deformation field, which can handle coarsely sampled and coplanar particle configurations. In this method liquids and deformable objects are uniformly represented and processed with SPH. Solenthaler et al. [7] also presented a formulation based on SPH which can handle density discontinuities at interfaces between multiple fluids correctly.

As an important particle-based method, SPH has been used for a wide range of applications in computer graphics. Müller et al. [8] showed that SPH could produce compelling fluid simulations at interactive rates. Up to now, researchers have used SPH to model such phenomena as incompressible flow[9] , solid-fluid coupling[3, 4] , and fluid-fluid interaction[10].

3 Deformable Solid-Fluid Interaction

3.1 Smoothed Particle Hydrodynamics

SPH is a particle-based model which was used for simulating fluid and deformable solid. It uses a set of particles as a discrete approximation of a continuum, expressing a field quantity *A(x)* by interpolating between the respective quantities around point x as follows

$$A(x) = \sum_i m_i \frac{A_i}{\rho_i} W(x - x_i, h) \tag{1}$$

where m_i is the mass of neighboring particle i, ρ_i its density, and x_i its position. $W(x, h)$ is a smooth kernel function with radius h.

When applied to fluid and deformable solid, each of the terms in the governing equations is expressed in Equation (1). Based on the work of Müller et al. [8] and Solenthaler et al. [4] , we compute the pressure and viscosity terms for fluid, and elastic force for solid as follows

Table 1 Attributes of each particle

Attribute	Description	Unit
T	particle type	1
m	mass	kg
V	volume	m^3
x	position	m
v	velocity	m/s
f	accumulated forces	N/m^3
ρ	density	kg/m^3
C	color attribute for surface tension	1

$$F_i^{pressure} = -\sum_j m_j \frac{P_i + P_j}{2\rho_j} \nabla W(x_i - x_j, h) \tag{2}$$

$$F_i^{vis\cos ity} = \mu \sum_j m_j \frac{v_j - v_i}{\rho_j} \nabla^2 W(x_i - x_j, h) \tag{3}$$

$$\begin{aligned} F_{ji}^{elastic} = & - \nabla u_j U_i = -2\bar{V}_j^2 (I + \nabla u_i^T)\sigma_i \\ & \nabla W(x_i - x_j, h) \end{aligned} \tag{4}$$

where μ is the viscosity coefficient, P_i the pressure, v_i the velocity, σ_i the stress, I the identity matrix, $\bar{V}$ the body volume, and $F_{ji}^{elastic}$ is the force particle i exerts on its jth neighbor.

3.2 Unified SPH Model for Interaction

By breaking down deformable solids and fluids into particles with various properties, we use unified SPH model to simulate deformable solids and fluids. Each particle individually carries many attributes summarized in Table 1.

During the interaction simulation, there are three types of forces to be calculated: force between fluid particles $F^{fluid-fluid}$, force between deformable solid particles $F^{solid-solid}$, and interaction force between deformable solid particles and fluid particles $F^{solid-fluid}$. We treat deformable solid as a special fluid constrained to solid deformation, thus the two-way interaction can be achieved directly by solving the governing equations of fluid. $F^{solid-fluid}$ is computed in the same way as $F^{fluid-fluid}$ which is the summation of $F^{pressure}$ and $F^{viscosity}$ solved by Equation (2) and (3). $F^{solid-solid}$ is solved using Equation (4).

However, the density calculated by Equation (1) cannot accurately represent sharp density changes when a particle has neighbor particles with different rest densities, because the kernel function smoothes the density. The falsified densities induce wrong pressure values close to the interface, leading to a spurious interface tension and a large gap. Based on the work of Solenthaler et al. [7], we introduce a new density model for deformable solid-fluid interaction with large rest density ratio

$$\tilde{\rho}(x_i) = m_i \sum_{j \in N_i^{neighbors}} W(x_i - x_j, h) \tag{5}$$

where $N_i^{neighbors}$ includes neighboring fluid particles and solid particles. The model makes each particle treat its neighbors as if they would have the same rest density and mass as itself. Thus, we get the new formulation for the pressure force and viscous force

$$F_i^{pressure} = -V_i \sum_j V_j \frac{\tilde{P}_i + \tilde{P}_j}{2\rho_j} \nabla W(r_{ij}, h) \tag{6}$$

$$F_i^{vis\cos ity} = V_i \sum_j \frac{\mu_i + \mu_j}{2} V_j (v_j - v_i) \nabla^2 W(r_{ij}, h) \tag{7}$$

where V is the volume of particle, $\tilde{P} = k(\rho - \rho_0)$, and k is a gas constant. With the modified density and pressure force equations we are able to eliminate spurious and unnatural interface tension effects which are present in the standard SPH method.

3.3 Surface Tension Model

Surface tension is important for interaction simulations, especially when we wish to synthesize small-scale fluid motions such as water drops flowing on a deformable solid. Müller et al. [8,10] proposed a surface tension model based on a smoothed color field C. The surface force is expressed as a volumetric force

$$F^{surface} = \alpha \kappa n = -\alpha \nabla^2 C \frac{n}{|n|} \tag{8}$$

where α is surface tension coefficient, $n = \nabla C / |\nabla C|$ the normal, and the curvature $\kappa = -\nabla^2 C / |n|$ is the divergence of the normal. It requires the calculation of curvature, which is difficult to predict, i.e. the divergence of the unit interface normal direction. In this paper, we present a novel surface tension method for deformable solid-fluid interaction with large density ratios. We introduce a new color function C as

$$C_i^j = \begin{cases} 1, & \text{if } j \text{ does not belong to the phase of } i \\ 0, & \text{if } j \text{ belong to the phase if } i \end{cases} \tag{9}$$

Physically, the interfacial motion is mainly driven by the material with large rest density. In order to reflect this behavior, we introduce a new density-weighted summation for the gradient of the color function

$$\nabla C_i = \frac{1}{V_i} \sum_j (V_i^2 + V_j^2) \frac{\rho_i}{\rho_i + \rho_j} C_i^j \nabla W(x_i - x_j, h) \tag{10}$$

To calculate the interface curvature, we use a reproducing divergence approximation. Starting from a Taylor series of a continuous vector field about the ith particle, we multiply the equation with the gradient of the kernel function and integrate over the entire domain. Neglecting the second and higher order terms, we obtain the summation form of the corrected gradient as

$$\nabla \phi_i = \left[\sum_j \phi_{ji} \otimes \nabla W(r_{ji}) V_j \right] \times \left[\sum_j r_{ji} \otimes \nabla W(r_{ji}) V_j \right]^{-1} \tag{11}$$

where $r_{ji} = x_j - x_i$. The denominator is a $d \times d$ matrix, and d is matrix dimensions. Computing the inverse matrix of the denominator and taking the trace of Equation (11), we find that the approximated divergence used to calculate the curvature can be written as

$$\nabla \cdot \phi_i = d \frac{\sum_j \phi_{ij} \cdot \nabla W(r_{ij}) V_j}{\sum_j |r_{ij}| \frac{\partial W}{\partial |r_{ij}|} V_j} \tag{12}$$

Using the above formulation in which only two simple summations are required, we can get a stable and accurate curvature of the interface.

4 Experiment Results

For validating the proposed interaction model, two experiments were conducted. The simulation and rendering parts of our system are implemented on a Microsoft Windows XP PC with dual Intel Core 2.8GHz CPUs, 2.0GB RAM. To generate fluid surfaces, the iso-surfaces of color fields[8] are extracted by using the Marching Cubes algorithm, and the resulting triangles were rendered with POV-Ray 3.6.

In Figure 1, our surface tension model is validated. The animation describes that a water stream falls onto an elastic duck. For the animation shown we used up to 8000 particles, the average CPU time per frame is 1.8 seconds. The rest density of the water particle is 1000 kg/m^3, and the density of the duck particle is 5000

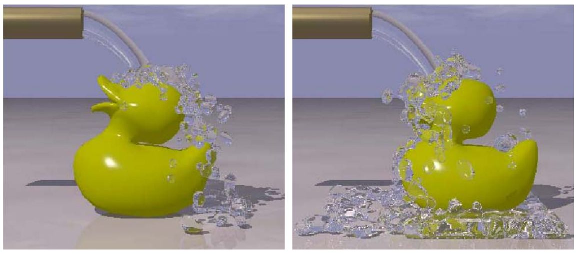

Fig. 1 A water stream falls onto a deformable duck

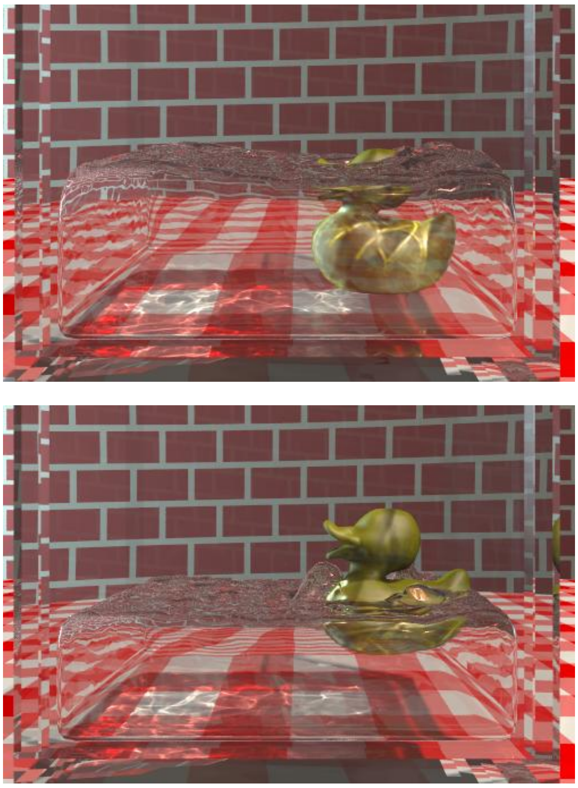

Fig. 2 Deformable solid-fluid interaction with large density ratio

kg/m^3. The duck causes splashes when it is battered by the water stream, and in the meantime, the water also makes the duck deform elastically. What is more, a variety of small-scale phenomena including the flow of water on solid surface and the formation of water droplets can be realistically simulated.

Figure 2 illustrates the density contrast two-way interaction between large amounts of water and an elastic duck inside a glass box. The water and the deformable duck are modeled with 22000 particles, the average CPU time per frame is 6.5 seconds. The first figure shows that a deformable duck with rest density 10000 kg/m^3 sinks into the water whose rest density is 1000 kg/m^3. Conversely, in the second figure the duck with rest density 100 kg/m^3 floats on the water.

Compared with other related methods[1,2,5] , the unified particle-based formulation eliminating the complex intermediate interface for deformable solid-fluid interaction is the remarkable advantage. This method achieves the comparable realistic simulation as in[3,4] , and the required CPU time has been decreased significantly.

5 Conclusion and Future Work

We have proposed a unified particle-based method to simulate deformable solid-fluid interaction. In the model we treat the deformable solid as a special fluid constrained to solid motion, so the two-way coupling between deformable solid and fluid is a straightforward extension of a multi-fluid solver. In order to strengthen interaction simulation realism, a discontinuous density model and a surface tension model with a stable and accurate surface curvature calculation are provided. The results show that our method is realistic and robust.

In the future, the method will be extended to simulate more complex interactions such as two-way coupling between fluid and porous or plastic solid.

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